

Fractals: a summary.

A fractal is "a rough or fragmented geometric shape" that is self similar – parts of the object look like the whole. The term "fractal" was coined by Mandelbrot - derived from the Latin fractus meaning "broken" or "fractured."

Typically a fractal often has the following features:

- It has a fine structure at arbitrarily small scales.
- It is too irregular to be easily described in traditional Euclidean geometric language.
- It is approximately self-similar

Because they appear similar at various levels of magnification, fractals are infinitely complex. Natural objects that are approximated by fractals to a degree include clouds, mountain ranges, lightning bolts, coastlines, snow flakes, some vegetables (eg cauliflower and broccoli).

A natural geometric invariant of a fractal is its *Hausdorff dimension* – an extension of the usual idea of dimension and can be fractional. It basically identifies the scaling properties of the object. I'll talk about this.

I'll also talk about a couple of examples:

Fractals in nature:



DLA (Diffusion Limited Aggregation – a model for how crystals form) and random walks.



(real thing: copper crystal)



(mathematical model)

Fractals in music – Bach's "The Art of Fugue" where simple patterns are repeated over and over at different scales and symmetries to produce a compelling piece of music.